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Nonlinear PI Controllers with Output Transformations

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Well-designed nonlinear proportional-integral (PI) controllers are successful for nonlinear dynamical processes like linear PI controllers are for linear processes. Two nonlinear blocks representing proportional and integral terms can be designed so that the linearized controllers perform the same as linear PI controllers for linearized processes at the given operating points. For some nonlinear processes, nonlinear blocks for nonlinear PI controllers can be singular at some operating points, and control performances can be poor for set points near those points. To mitigate such disadvantages, new nonlinear PI controllers that introduce output transformations are proposed. Several examples are given, showing the performance of the proposed nonlinear PI controllers.
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Introduction

Proportional-integral (PI) controllers are very successful for controlling many processes and can even be effective for nonlinear processes.[1] When operating points are not changing, linear PI controllers designed using linearized models for given set points[2] can be used. A large number of tuning rules are available for linear PI controllers.[3–6] When operating points are changing for nonlinear processes, linearized model parameters will also change. For nonlinear processes with mild changes of linearized model parameters, robust linear PI controllers designed based on the average or worst-case model parameters can be used. For nonlinear processes with highly nonlinear terms whose linearized model parameters change significantly as the operating points change, PI controller parameters should be changed for better closed-loop performances. Parameterized control systems with changing operating points have long been studied in the categories of "gain scheduling"[7] or "extended linearization".[8,9] Linear and nonlinear PI controllers with constant or gain-scheduled parameters have been used for various processes, for example, fuel cell processes,[10] bio processes,[11] batch processes,[12] and real-time optimization.[13]

There have been several types of nonlinear PI controllers that do not use explicit gain scheduling parameters. Rugh[14] proposed nonlinear PI controllers that track Ziegler–Nichols PI control systems for changing set points. Lee et al.[15] proposed nonlinear control systems based on parameterized first-order plus time delay process models. Wright et al.[16] proposed nonlinear PI controllers based on a nonlinear first-order model, whose linearized closed-loop systems are the same as internal model control (IMC) PI control systems for given operating points. Brendel et al.[17] proposed nonlinear PI controllers based on the nonlinear first-order plus time delay model. The Rugh and Wright et al. methods belong to the extended linearization method, and stability of a nonlinear PI control system can be guaranteed for slowly varying operating points.[8]

Nonlinear PI controllers have two blocks for proportional and integral terms. For some nonlinear processes, they can be nonanalytic in the range of operating points. Control performances of nonlinear PI controllers can be poor at set points near such nonanalytic points. To relieve such drawbacks, nonlinear PI controllers with output transformations are proposed. Through simulations, the use of proposed nonlinear PI controllers provides far better control performances for set points near nonanalytic points, compared to existing nonlinear PI controllers. For set points far from nonanalytic points, all nonlinear PI controllers based on the extended linearization method have very similar control performances.

Output transformations can be found in various processes such as pH neutralization[18,19] and high purity distillation.[20] The pH neutralization process dynamics are described well by the Wiener model and can be controlled through an output transformation. Here, with introducing the output transformation, the integral block in the nonlinear PI controller block is made linear. When processes are controlled loosely and the closed-loop responses are slow, there is less emphasis placed on the proportional mode. A two time scale analysis applied to slowly controlled systems proves this.[21,22] In these cases, the proportional parts can be approximated by linear ones, and nonlinear PI controllers can be greatly simplified. Additionally, problems due to nonanalytic points can be resolved.

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Previous Nonlinear PI Controller Design Methods

Consider a nonlinear single-input single-output process

$$\dot{x} = f(x) + g(x)u \qquad y = h(x) \tag{1}$$

where x is a state vector and u and y are the scalar input and output variables, respectively. It is assumed that the static map between the input u_s and the output y_s is given by a strictly increasing function

$$y_s = q(u_s) \tag{2}$$

The subscript s denotes the steady-state value. At a given steady-state input u_s (corresponding state variables x_s and output y_s), this nonlinear process can be approximated by a linear system

$$Y(s) = \frac{k(u_s)e^{-\theta s}}{\tau(u_s)s+1}U(s) \tag{3}$$

where $k(u_s)$, $\tau(u_s)$, and θ are the steady-state gain, time constant, and time delay of the linearized process, respectively. When variables are maintained near their steady-state values, a control system based on the linear model of Eq. 3 can be used. For example, the IMC method can be used to design a PI controller for the linearized process of Eq. 3

$$U(s) = \left(k_c + \frac{k_I}{s}\right)E(s), \quad E(s) = Y_s(s) - Y(s)$$
$$k_c = \frac{\tau(u_s)}{\lambda k(u_s)} = \frac{\tau(q^{-1}(y_s))}{\lambda k(q^{-1}(y_s))} \tag{4}$$
$$k_I = \frac{1}{\lambda k(u_s)} = \frac{1}{\lambda k(q^{-1}(y_s))}$$

This design method of a parametric control system is known as "extended linearization method" or "gain scheduling method."[7–9] Analytic controllers realizing the above parametric controller of Eq. 4 can be expressed as

$$\dot{z} = \frac{1}{\lambda}e$$
$$u = \frac{\xi_1(z,y)}{\lambda}e + \xi_2(z,y), \quad e = y_s - y \tag{5}$$

where nonlinear functions of ξ_1 and ξ_2 represent the proportional and integral terms, respectively. When the nonlinear PI controller of Eq. 5 is equivalent to the linearized PI controller of Eq. 4 for the whole operating points of u_s and y_s, local stability of the closed-loop system is guaranteed for slowly varying u_s (or correspondingly a slowly varying set point y_s).[8] Several nonlinear PI controllers in the form of Eq. 5 have been available.

Rugh method

The control system of Eq. 4 can show jumps in the manipulated variable for a change of the set point y_s. For a bumpless transfer, Rugh has proposed a nonlinear PID controller based on the Ziegler–Nichols tuning rule. Applying the Rugh method to the IMC tuning of Eq. 4, we have

$$\dot{\zeta} = k_I(\zeta)e = \frac{1}{\lambda k(\zeta)}e$$
$$u = k_c(\zeta)e + \zeta = \frac{\tau(\zeta)}{\lambda k(\zeta)}e + \zeta \tag{6}$$

This controller is linearized as

$$\dot{\tilde{\zeta}} = \frac{1}{\lambda k(\zeta_s)}e, \quad \tilde{u} = \frac{\tau(\zeta_s)}{\lambda k(\zeta_s)}e + \tilde{\zeta} \tag{7}$$

where tilde denotes the deviation variable. As the steady-state value ζ_s is equal to u_s, the linearized controller of Eq. 7 is the same as Eq. 4.

For the nonlinear PI controller in the form of Eq. 5, we apply the transformation

$$z = q(\zeta) \tag{8}$$

Then, the nonlinear PI controller of Eq. 6 becomes (as $q'(\zeta) = k(\zeta)$)

$$\dot{z} = k(\zeta)\dot{\zeta} = \frac{1}{\lambda}e$$
$$u = \frac{\tau(q^{-1}(z))}{\lambda k(q^{-1}(z))}e + q^{-1}(z) \tag{9}$$

WKK method

For a nonlinear first-order model

$$A(y)\dot{y} + B(y) = u \tag{10}$$

Wright, Kravaris, and Kazantzis[16] proposed a nonlinear PI controller

$$\dot{z} = \frac{1}{\lambda}e$$
$$u = A(z)\frac{e}{\lambda} + B(z) \tag{11}$$

For the process of Eq. 10, we have

$$u_s = q^{-1}(y_s) = B(y_s), \quad k = \frac{1}{B'(y_s)}, \quad \tau = \frac{A(y_s)}{B'(y_s)} \tag{12}$$

At the steady state, z_s becomes y_s and it can be shown that the nonlinear PI controller of Eq. 11 has the same linearized transfer function as Eq. 4 for a given y_s. The nonlinear controller of Eq. 11 is equivalent to that of Eq. 9 (equivalently, the Rugh controller of Eq. 6). When implemented in the sampled system, different responses between nonlinear PI controllers of Eqs. 6 and 11 can be obtained due to sampling effects.

Nonlinear PI controllers can be applied to processes with time delays; see Table 1.

BOD method

For an approximate model of Eq. 10, Brendel, Ogunnaike, and Dhurjati[17] proposed a nonlinear PI controller

$$u = \frac{A(y)}{\lambda}\left(e + \frac{B(y)}{yA(y)}z\right) = \frac{A(y)}{\lambda}e + \frac{B(y)}{y}z \tag{13}$$

Unlike the previous two methods of Rugh and WKK, the linearized transfer function of Eq. 13 is different from Eq. 4.

Example 1: (Wiener process)

Consider the first-order process with time delay

$$\dot{x} = -x + u(t-\theta) \qquad y = x^3 \tag{14}$$

For this process, we have $y_s = u_s^3$, $k = 3u_s^2 = 3y_s^{2/3}$, $\tau = 1$. Hence, the Rugh PI controller is

Table 1. Nonlinear PI Controllers

Process Model	Method	Controller	Comments
Steady-state map: $y_s = q(u_s) = \int k(u)du$	Rugh	$\dot{z} = \frac{1}{\lambda k(z)} e$, $u = \frac{\tau(z)}{\lambda k(z)} e + z$	$z_s = u_s$
Linearized model: $\frac{Y(s)}{U(s)} = \frac{k(u_s)e^{-\theta s}}{\tau(u_s)s+1}$	WKK	$\dot{z} = \frac{1}{\lambda} e$, $u = \frac{\tau(q^{-1}(z))}{\lambda k(q^{-1}(z))} e + q^{-1}(z)$	$z_s = y_s$
	Proposed 1	$\dot{z} = \frac{1}{\lambda} \hat{e}$, $u = \frac{\tau(z)}{\lambda} \hat{e} + z,\ \ \hat{e} = q^{-1}(y_s) - q^{-1}(y)$	$z_s = u_s = q^{-1}(y_s)$
$A(y)\dot{y} + B(y) = u(t-\theta)$	WKK	$\dot{z} = \frac{1}{\lambda} e$, $u = A(z)\frac{e}{\lambda} + B(z)$	$z_s = y_s$
	BOD	$\dot{z} = \frac{1}{\lambda} e$, $u = \frac{A(y)}{\lambda} e + \frac{B(y)}{y} z$	$z_s = y_s$
	Proposed 2	$\dot{z} = \frac{1}{\lambda} \hat{e}$, $u = \frac{A(y)}{\lambda B'(y)} \hat{e} + z,\ \ \hat{e} = B(y_s) - B(y)$	$z_s = u_s = B(y_s)$

(u_s and y_s are the steady-state values of input and output, respectively, and $e = y_s - y$. λ is the design parameter representing the closed-loop time constant.)

$$\dot{\zeta} = \frac{1}{3\lambda\zeta^2} e, \quad u = \frac{1}{3\lambda\zeta^2} e + \zeta \tag{15}$$

Equation 14 can be rewritten as

$$\dot{y} = 3x^2\dot{x} = -3y + 3y^{2/3}u(t-\theta) \tag{16}$$

Hence $A(y) = y^{-2/3}/3$ and $B(y) = y^{1/3}$. The WKK controller has

$$\dot{z} = \frac{1}{\lambda} e, \quad u = \frac{1}{3\lambda z^{2/3}} e + z^{1/3} \tag{17}$$

When these controllers are applied for the set point of $y_s = 0$, ζ, and z go to zero and both proportional and integral terms can be very large, resulting in highly oscillatory responses. For $\theta = 1$ and $\lambda = 2$, closed-loop responses are shown in Figure 1. Here, we use $y^{1/3} = \text{sign}(y) \cdot |y|^{1/3}$.

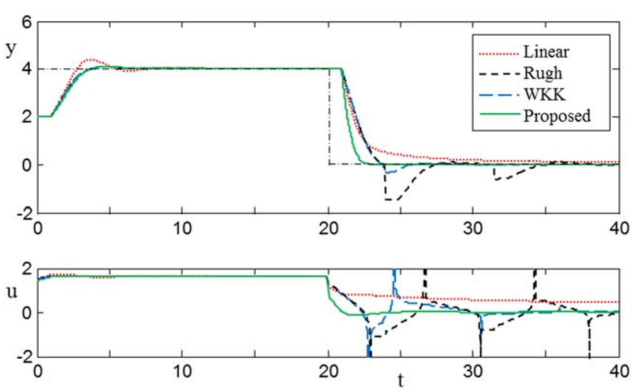

Figure 1. Closed-loop responses for Example 1 ($\theta = 1$, $\lambda = 2$).

Linear controller parameters are $k_C = 0.1$ and $\tau_I = 1$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

The BOD controller is

$$u = \frac{1}{3\lambda y^{2/3}} e + \frac{1}{y^{2/3}} z \tag{18}$$

The closed-loop system with this controller is unstable for small y_s.

Proposed Method

As shown in Example 1, all three methods (Rugh, WKK, and BOD) show poor closed-loop responses at the operating points of small y_s. Here, we propose a nonlinear PI control method to resolve such drawbacks, (Controller 1)

$$\dot{v} = \frac{1}{\lambda} \hat{e}$$
$$u = \frac{\tau(v)}{\lambda} \hat{e} + v, \quad \hat{e} = q^{-1}(y_s) - q^{-1}(y) \tag{19}$$

At steady state, $v_s = u_s$. The nonlinear PI controller of Eq. 19 is linearized as

$$\dot{\tilde{v}} = \frac{1}{\lambda} \hat{e}, \quad \tilde{u} = \frac{\tau(u_s)}{\lambda} \hat{e} + \tilde{v}, \quad \hat{e} = -\frac{1}{q'(u_s)} \tilde{y} = \frac{1}{k(u_s)} e \tag{20}$$

It is evident that the nonlinear controller of Eq. 19 has the same linearized transfer function as Eq. 4 for a given u_s. For the process of Eq. 10, the proposed method is given in Table 1. Figure 2 shows the block diagram of this nonlinear PI control system.

When $\tau(q^{-1}(y_s))$ is available instead of $\tau(u_s)$, we can use (Controller 2)

$$u = \frac{\tau(q^{-1}(y))}{\lambda} \hat{e} + v \tag{21}$$

When $\tau(u_s)$ has less/limited variability or processes are controlled loosely, we use a constant proportional gain (Controller 3)

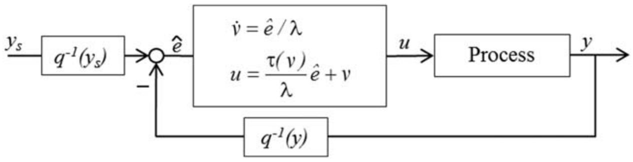

Figure 2. Schematic diagram of the proposed nonlinear PI control system (Proposed 1).

$$u = k_c\hat{e}+v \tag{22}$$

because the proportional gain does not significantly affect control performances. This simplification is based on two-time scale analysis of closed-loop systems with loosely tuned integral controllers.[14,15]

The following observations are noted:

1. Linearized controllers for Eqs. 19 and 21 are the same as Eq. 4, the IMC controller for the linearized process.
2. Nonlinear functions with singular points for the internal variable v can be removed in the nonlinear PI controller of Eq. 19 or Eq. 21.
3. For nonlinear PI controllers of Eq. 21, we can use the velocity form of a PI controller

$$u_{k+1} = u_k + \frac{\tau(q^{-1}(y_{k+1}))}{\lambda}\hat{e}_{k+1} - \frac{\tau(q^{-1}(y_k))}{\lambda}\hat{e}_k + \frac{1}{\lambda}\hat{e}_{k+1}T \tag{23}$$

Here, T is the sampling time and the subscript k means the kth instant. With this velocity form, the anti-reset-windup function can be implemented simply by clipping u_k.[3]

For Example 1, the proposed nonlinear PI controller is

$$\dot{v} = \frac{1}{\lambda}\hat{e},\ \ u = \frac{1}{\lambda}\hat{e}+v,\ \ \hat{e} = {y_s}^{1/3}-y^{1/3} \tag{24}$$

There are no nonlinear functions showing very large values for whole operating points of u_s and y_s. Excellent closed-loop responses as shown in Figure 1 can be obtained.

Simulations

Example 2: (pH process)

Consider a pH neutralization process where the influent stream of the acetic acid (CH_3COOH) is titrated by a strong base of sodium hydroxide (NaOH) in the continuous stirred tank reactor. It is assumed that the mixing is perfect and the acid-base reaction is very fast. The mathematical model equations are given as follows[18,19]

$$\begin{aligned} V\dot{c}_a &= -(F+u)c_a + Fc_{aF} \\ V\dot{c}_b &= -(F+u)c_b + u\,c_{bT} \\ -\frac{c_a}{1+10^{-\mathrm{pH}}/K_a} &+ c_b + 10^{-\mathrm{pH}} - K_w/10^{-\mathrm{pH}} = 0 \\ y &= \mathrm{pH} \end{aligned} \tag{25}$$

The process parameters used are: V is reactor volume (2 L); c_a, c_{aF} is acid (CH_3COOH) concentration in the reactor solution and feed stream ($c_{aF} = 0.02$ M); K_a is dissociation constant of CH_3COOH (1.8E−5); F is feed flow rate (0.1 L/min); c_b, c_{bT} is base (NaOH) concentration in the reactor solution and titrating stream ($c_{bT} = 0.2$ M); u is titrating stream flow rate (manipulated variable); K_w is ion product of water (1.0E−14); pH is concentration of hydrogen ion in the reactor solution (controlled variable).

For $u \ll F$, the process can be approximated as[19]

$$V\dot{x} = -Fx + u(t-\theta)$$
$$x = \phi(y) = \frac{-\frac{c_{aF}}{1+10^{-y}/K_a} + 10^{-y} - \frac{K_w}{10^{-y}}}{-\frac{c_{aF}}{1+10^{-y}/K_a} - c_{bT}} \tag{26}$$

Here, the time delay $\theta = 0.2$ is introduced to represent ignored dynamics such as mixing. The pH process of Eq. 26 can be rewritten as

$$V\phi'(y)\dot{y} + F\phi(y) = u(t-\theta) \tag{27}$$

The WKK controller is

$$\dot{z} = \frac{1}{\lambda}e,\ \ u = \frac{V\phi'(z)}{\lambda}e + F\phi(z),\ \ e = y_s - y \tag{28}$$

The BOD controller is

$$u = \frac{V\phi'(y)}{\lambda}e + \frac{F\phi(y)}{y}z \tag{29}$$

The proposed controller is

$$\dot{v} = \frac{1}{\lambda}\hat{e},\ \ u = \frac{V}{\lambda F}\hat{e} + v,\ \ \hat{e} = F\phi(y_s) - F\phi(y) \tag{30}$$

Figure 3 shows closed-loop responses. The sampling time is set to 0.05 min, θ = 0.2 and $\lambda = 5$. For this process, $\phi'(y)$ can be large for y near 14. For y_s = 9, z for the WKK controller can be near 14, resulting in vigorous control actions (see Figure 3). Because y does not go near 14, the BOD controller shows excellent closed-loop responses for y_s between 5 and 9. For y_s near 14, the BOD controller suffers from vigorous control actions. The proposed nonlinear PI controller has no functions with large values throughout the whole operating points. As shown in Figure 3, excellent closed-loop responses of the nonlinear PI control system can be obtained.

Example 3: (Polymerization reactor)

Consider the process in Brendel et al.[17]

$$\begin{aligned} \dot{x}_1 &= 10(6-x_1) - 2.4568x_1\sqrt{x_2} \\ \dot{x}_2 &= -10.1022x_2 + 1.3432(1+u) \\ \dot{x}_3 &= 0.0024121x_1\sqrt{x_2} + 0.112191x_2 - 10x_3 \\ \dot{x}_4 &= 245.978x_1\sqrt{x_2} - 10x_4 \\ y &= \frac{x_4}{25000.5x_3} - 1 \end{aligned} \tag{31}$$

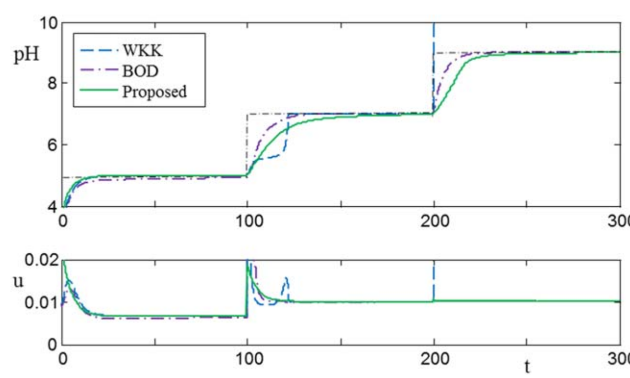

Figure 3. Closed-loop responses for Example 2.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

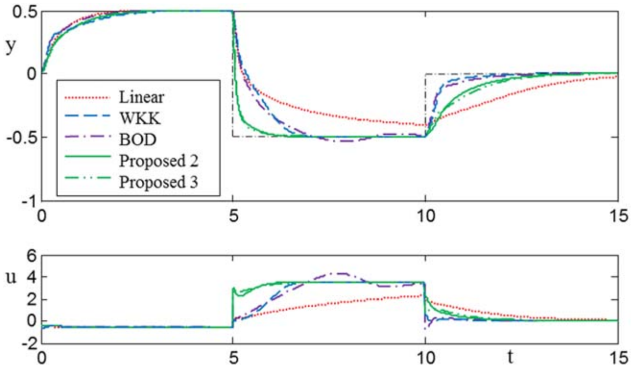

Figure 4. Closed-loop responses for Example 3.

Linear controller parameters are $k_C = -1.083$ and $\tau_I = 0.3454$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

This process is a unique case of a free-radical polymerization reactor model given by Daoutidis et al.[23] For this process, for the operating range between $y=-0.4$ and 0.4, they obtained an approximate first-order model of[23]

$$\dot{y} = -(2.895y + 2.355y^2) - (1.385 + 2.943y + 1.224y^2)u \quad (32)$$

With a first-order model of $A(y) = -1/(1.385 + 2.943y + 1.223y^2)$ and $B(y) = -(2.895y + 2.355y^2)/(1.385 + 2.943y + 1.223y^2)$, nonlinear PI controllers can be obtained as follows.

The WKK controller is

$$\dot{z} = \frac{1}{\lambda}e, \quad u = \frac{e + \lambda(2.895z + 2.355z^2)}{-\lambda(1.385 + 2.943z + 1.224z^2)} \quad (33)$$

The BOD controller is

$$u = \frac{e + \lambda(2.895 + 2.355y)z}{-\lambda(1.385 + 2.943y + 1.224y^2)} \quad (34)$$

For the proposed controller 2

$$\dot{v} = \frac{1}{\lambda}\hat{e}$$

$$u = \frac{1.385 + 2.943y + 1.224y^2}{\lambda(4.0096 + 6.5233y + 3.3873y^2)}\hat{e} + v$$

$$\hat{e} = -\frac{2.895y_s + 2.355y_s^2}{1.385 + 2.943y_s + 1.224y_s^2} + \frac{2.895y + 2.355y^2}{1.385 + 2.943y + 1.224y^2} \quad (35)$$

Proposed controller 3 with a constant proportional gain is

$$u = \frac{1.385}{4.0096\lambda}\hat{e} + v \quad (36)$$

These nonlinear PI controllers are applied to the process of Eq. 31.

Figure 4 shows closed-loop responses for set points of $y_s = 0.5$, -0.5, and 0. The sampling time is set to 0.002 and $\lambda = 0.5$. The WKK and BOD controllers show somewhat sluggish responses for $y_s = -0.5$. The proposed controller 2 shows excellent responses for all three set points. The proposed controller 3 with a constant proportional gain also shows very similar responses to other nonlinear PI controllers with variable proportional gains. The WKK and BOD controllers have nonanalytic proportional gains at $y_s = -0.6421$ because it is a root of $1.224y_s^2 + 2.943y_s + 1.385 = 0$. The three proposed controllers have no such nonanalytic point for the internal variable v. For y_s near this nonanalytic point, the WKK and BOD controllers can show overactive control behavior.

Example 4: (Liquid level system)

A liquid level system shown in Figure 5 is considered with process model

$$A\dot{h} = -a\sqrt{2gh}\,u(t-\theta) + q_{in} \quad (37)$$

where A is the tank cross-sectional area, a is the orifice area, h is the water level in the tank, q_{in} the influent flow rate, and g is the gravitational acceleration. The effluent flow is controlled by the solenoid valve with pulse width modulation signal ($u(t-\theta)$). A time delay θ is added (artificially) to make the control task more difficult. Normalizing the time and other variables in Eq. 37, we use the process model

$$-\frac{1}{\sqrt{y}}\dot{y} + \frac{1}{\sqrt{y}} = u(t-\theta) \quad (38)$$

The WKK controller is

$$\dot{z} = \frac{1}{\lambda}e, \quad u = -\frac{1}{\lambda\sqrt{z}}e + \frac{1}{\sqrt{z}} \quad (39)$$

For this process, we have $y_s = 1/u_s^2$, $k = -2/u_s^3 = -2y_s^{1.5}$, $\tau = 2y_s = 2/u_s^2$. The proposed controller 1 is

$$\dot{v} = \frac{1}{\lambda}\hat{e}, \quad u = \frac{2}{\lambda v^2}\hat{e} + v, \quad \hat{e} = 1/\sqrt{y_s} - 1/\sqrt{y} \quad (40)$$

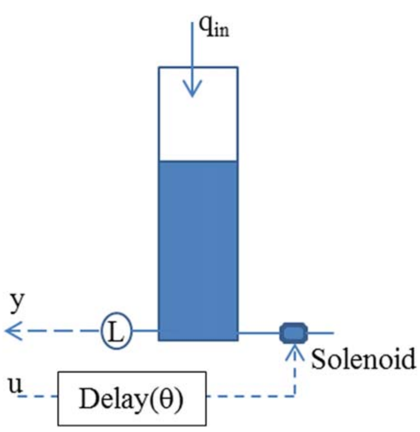

Figure 5. Liquid level system.

[Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

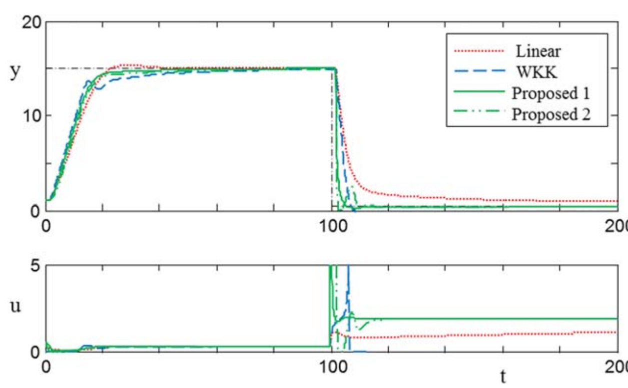

Figure 6. Closed-loop responses for Example 4.

Linear controller parameters are $k_C = -0.05$ and $\tau_I = 0.1$. [Color figure can be viewed in the online issue, which is available at wileyonlinelibrary.com.]

Proposed controller 2 is

$$u=\frac{2y}{\lambda}\hat{e}+v \tag{41}$$

Figure 6 shows closed-loop responses. The sampling time is set to 0.1, θ = 2, λ = 4, and u is bounded between 0 and 10. The WKK controller shows vigorous control actions for y_s = 0.3. The internal variable z becomes very small and even goes to negative values, causing u to have imaginary numbers. Clipping the variable z is required for the WKK controller. The proposed controller 1 of Eq. 40 can have large values due to the term v^{-2} for a large y_s. However, limits on u prevent vigorous control actions. The proposed controller of Eq. 41 is free from nonanalytic nonlinearities.

Conclusion

For some nonlinear processes, nonlinear PI controllers can have nonanalytic points in the operating region. For set points near such nonanalytic points, control performances can be poor. To relieve such drawbacks, several nonlinear PI controllers which utilize output transformations are proposed and tested using various simulated process examples.

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